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Common Fixed Point Theorems for Multivalued Compatible Maps in IFMS

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Abstract-The aim of this paper is to obtain the notion of multivalued weakly compatible (mwc) maps and prove common fixed point theorems for single and multi valued maps by using a contractive condition of integral type in intuitionistic fuzzy metric spaces.

Index Terms- Fixed Points , intuitionistic fuzzy metric space, multivalued weakly compatible maps, compatible maps.

1. Introduction and Preliminaries

A fundamental result in fixed point theory is intuitionistic fuzzy metric spaces which is stated in theorem Through out the paper X will represent the intuitionistic fuzzy metric space (X, M, N, *, ◊) and CB(X) , the set of all non-empty closed and bounded sub-sets of X . For A, B ∈ CB(X)and for every t>0,

denote H(A, B, t)=sup{M(a, b, t);a ∈ A, b ∈ B} and H(A, B, t)=inf{N(a, b, t);a ∈ A, b ∈ B}

and δM(A, B, t)=Inf{ M(a, b, t);a ∈ A, b ∈ B},

δN(A, B, t)=sup{N(a, b, t);a ∈ A, b ∈ B}

If A consists of a single point a, we write

δM(A, B, t)= δM(a, B, t) and δN(A, B, t)= δN(a, B, t). If B also consists of a single point b, we write

δM(A, B, t)= M(A, B, t) and δN(A, B, t)= N(A, B, t)

It follows immediately from definition that

δM(A, B, t)= δM(B, A, t)≥0 and

δN(A, B, t)= δN(B, A, t)≥0

δM(A, B, t)=1 ⇔ A=B={a}

δN(A, B, t)=0 ⇔ A=B={a} for all A,B ∈ CB(X)¸

Definition: Maps A :X →X and B: X→ CB (X) are said to be multivalued weakly compatible (mwc) if there exists some point x ∈ X such that

Ax ∈ Bx and ABx ⊆ BAx.

Clearly weakly compatible maps are multivalued weakly compatible (mwc).

2. Main Result

Now, we prove our main result.

Theorem 1. Let (X,M, N, *, ◊) be a complete intuitionistic fuzzy metric space with continuous t-norm * and continuous t-corm ◊ defined by t*t=t and (1 - t) ◊ (1 - t) ≤ (1 - t) for all t ∈ [0, 1] such that, A :X →X and B: X→ CB (X) be single and multi valued mappings respectively such that the maps (A,S) and (B,T) are (mwc) and satisfy the inequality for all x, y ∈ X where φ :[0,1]→[0,1]

$$\int_0^{\delta M(Sx,Ty,kt)} \varphi(t)dt > \int_0^{m(x,y,t)} \phi(t)dt \qquad (1.1)$$

where

m(x, y, t)=min{[H(Ax, Sx, t)+H(By, Ty,t)], [M(Ax, By, t)*H(By, Ty, t)*H(By, Sx,(2-α)t)],
[H(Ax, Ty, αt)*H(By, Ty, t)*H(Sx, By, t)]}

$$\int_0^{\delta N(Sx,Ty,kt)} \varphi(t)dt < \int_0^{n(x,y,t)} \phi(t)dt \qquad (1.2)$$

where

n(x, y, t)=max{[H(Ax, Sx, t)+H(By, Ty,t)], [N(Ax, By, t)◊H(By, Ty, t)◊H(By, Sx,(2-α)t)],

[H(Ax, Ty, αt)◊H(By, Ty, t)◊H(Sx, By, t)]}

is a function which is sum able, Lebseque integrable, non-negative and such that

$$\int_0^{\epsilon} \phi(t)dt > 0$$

for each ε > 0 .for every x, y ∈ X and t > 0,

α ∈ (0,2). Then A, B, S and T have unique common fixed point in X.

Proof. Since the pairs (A, S) and (B,T) are occasionally weakly compatible(mwc)maps, therefore, there exist two elements , u v in X such that Au ∈ Su ¸ ASu ⊆ SAu and Bv ∈ Tv , BTv ⊆ TBv .First we prove that Au =Bv. As Au ∈ Su so AAu ⊂ ASu ⊂ SAu , Bv ∈ Tv ¸ so BBv ⊂ BTv ⊂ TBv and hence

M(A2u,B2v,t)≥δM (SAu,TBv, t), N(A2u,B2v,t)≤δN (SAu,TBv, t) and if Au≠ Bv then

δM (SAu,TBv, t)< 1 , δN (SAu,TBv, t)<1 .

Using (1.2) for x = Au, y= Bv

m(Au, Bv, t)=min{[H(AAu, SAu, t)+H(B Bv, TBv,t)], [M(AAu, B Bv, t)*H(B Bv, T Bv, t)*H(B Bv, SAu,(2-α)t)],[H(AAu, TBv, αt)*H(BBv, TBv, t)*H(SAu, BBv, t)]}

≥min{[M(AAu, SAu,t)+M(B Bv, TBv,t)], [M(AAu,BBv,t)*M(B Bv, T Bv, t)* M(BBv,SAu,(2-α)t)],[M(AAu, TBv, αt)*M(BBv, T Bv, t)*M(SAu, BBv, t)]} (1.3)

n(Au, Bv, t)=max{[H(AAu, SAu, t)+ H(BBv, TBv,t)],[N(AAu, BBv, t)◊H(BBv,TBv, t)◊H(B Bv, SAu,(2-α)t)], [H(AAu, TBv, αt)◊H(BBv, T Bv, t)◊H(SAu, BBv, t)]}

≤ max{[N(AAu, SAu, t)+N(B Bv, T Bv,t)], [N(AAu, B Bv, t)◊N(B Bv, TBv, t) ◊N(B Bv, SAu,(2-α)t)], [N(AAu, T Bv, αt)◊N(B Bv, T Bv, t)◊N(SAu, BBv, t)]} (1.4)

Since, * and ◊ is continuous , letting α→1 in (1.3) and (1.4), we get

m(Au, Bv, t) ≥ min{[M(A2u, SAu, t)+ M(B 2v, TBv,t)], [M(A2u, B 2v, t)* M(B 2v, T Bv, t)*M(B 2v, SAu, t)], [M(A2u, T Bv, t)*M(B 2v, T Bv, t) *M(SAu, B2v, t)]}

≥ min{[1+1], [δM(SAu, TB v, t)*1* δM(TB v, SAu, t)],[δM(SAu, TBv, t)*1 * δM(SAu, TBv, t)]} = δM(SAu, T Bv, t) (1.5)

n(Au, Bv, t) ≤max{[N(A2u, SAu, t)+ N(B 2v, TBv,t)], [N(A2u, B 2v, t) ◊N(B 2v, T Bv, t) ◊N(B 2v, SAu, t)], [N(A2u, T Bv, t) ◊ N(B 2v, T Bv, t) ◊N(SAu, B2v, t)]}

≤max {[0+0], [δN(SAu, TB v, t) ◊0 ◊δN(TB v, SAu, t)], [δN(SAu, T Bv, t) ◊0 ◊ δN(SAu, T Bv, t)]} = δN(SAu, T Bv, t) (1.6)

From (1.1) and (1.5) , (1.2) and (1.6) we have

$$\int_0^{\delta M(SAu,TBv,kt)} \varphi(t)dt > \int_0^{m(Au,Bv,t)} \phi(t)dt \geq \int_0^{\delta M(SAu,TBv,t)} \phi(t)dt$$

$$\int_0^{\delta N(SAu,TBv,kt)} \varphi(t)dt < \int_0^{n(Au,Bv,t)} \phi(t)dt \leq \int_0^{\delta N(SAu,TBv,t)} \phi(t)dt$$

, a contradiction.

Hence Au= Bv .

Also M(A2u, Bu, t)≥δ M (SAu, Tu, t),

N(A2u, Bu, t)≤δ N (SAu, Tu, t),

M(A2u, Tu, t)≥δ M (SAu, Tu, t),

N(A2u, Tu, t)≤δ N (SAu, Tu, t),

Now, we claim that Au= u . It not, then

δ M (SAu, Tu, t)<1, δ N (SAu, Tu, t)<1

Considering (1.1) and (1.2) for Au =x, u= y , α=1

m(Au, u, t)=min{[H(AAu, SAu, t)+H(Bu, Tu,t)], [M(AAu, Bu, t)*H(Bu, Tu, t)*H(Bu, S Au, t)], [H(AAu, Tu, t)*H(Bu, Tu, t)*H(S Au, Bu, t)]}

≥min{[M(A 2u, SAu, t)+M(Bu, Tu, t)], [M(A 2u, Bu, t)*M(Bu, Tu, t)*M(Bu, S Au, t)] [M(A 2u, Tu, t)*M(Bu, Tu, t)*M(S Au, Bu, t)]}

≥min{[1+1], [δM(SA u, Tu, t)*1*δ M(Tu, S Au, t)], [δ M(SA u, Tu, t)*1*δ M(S Au, Tu, t)]}

m(Au, u, t)≥ δM(SA u, Tu, t) (1.7)

n(Au, u, t)=max{[H(A Au, SAu, t)+H(Bu, Tu,t)], [N(A Au, Bu, t)◊H(Bu, Tu, t)◊H(Bu, S Au, t)], [H(A Au, Tu, t)◊H(Bu, Tu, t)◊H(S Au, Bu, t)]}

≤max{[N(A 2u, S Au, t)+N(Bu, Tu,t)],

From (1.1) and (1.7), (1.2) and (1.8) we have

$$\int_0^{\delta M(SAu,Tu,kt)} \varphi(t)dt > \int_0^{m(Au,u,t)} \phi(t)dt \geq \int_0^{\delta M(SAu,Tu,t)} \phi(t)dt$$

$$\int_0^{\delta N(SAu,Tu,kt)} \varphi(t)dt < \int_0^{n(Au,u,t)} \phi(t)dt \leq \int_0^{\delta N(SAu,Tu,t)} \phi(t)dt$$

which is again a contradiction and hence A=u .

Similarly, we can get Bv= v.Thus A, B, S and T have a common fixed point in X.For uniqueness let u ≠u ‘, be another fixed point of A, B, S and T, then (1.1) and (1.2) gives m(u, u', t)=min{[H(Au, Su, t)+H(Bu', Tu',t)], [M(Au, Bu', t)*H(Bu', Tu', t)*H(Bu', Su,(2-α)t)], [H(Au, Tu', αt)*H(Bu', Tu', t)*H(Su, Bu', t)]}

n(u, u', t)=max{[H(Au, Su, t)+H(Bu', Tu',t)], [M(Au, Bu', t)*H(Bu', Tu', t)*H(Bu', Su,(2-α)t)], [H(Au, Tu', αt)*H(Bu', Tu', t)*H(Su, Bu', t)]}

Letting α→1

m(u, u', t)=min{[δ M(Au, Su, t)+δ M (Bu', Tu',t)], [M(Au, Bu', t)* δ M (Bu', Tu', t)* δ M (Bu', Su, t)], [δ M (Au, Tu', t)* δ M (Bu', Tu', t)* δ M (Su, Bu', t)]}

m(u, u', t)=min{[1+1], [M(Su, Tu', t)* 1* δ M (Tu', Su, t)], [δ M (Su, Tu', t)* 1* δ M (Su, Tu', t)]}

m(u, u', t)= δ M (Su, Tu', t) (1.9)

n(u, u', t)=max{[δ N(Au, Su, t)+δ N (Bu', Tu', t)], [N(Au, Bu', t)◊ δ N (Bu', Tu', t)◊δ N (Bu', Su, t)], [δ N (Au, Tu', t)◊ δ N (Bu', Tu', t)◊ δ N(Su, Bu', t)]}

n(u, u', t)=max{[0+0], [N(Su, Tu', t)◊ 0◊ δ N (Tu', Su, t)],[δ N(Su, Tu', t)◊0◊ δ N (Su, Tu', t)]}

n(u, u', t)= δ N (Su, Tu', t) (1.10)

Again from (1.1) and (1.9), (1.2)and (1.10) we obtain

$$\int_0^{\delta M(Su,Tu',kt)} \varphi(t)dt > \int_0^{m(u,u',t)} \phi(t)dt \geq \int_0^{\delta M(Su,Tu',t)} \phi(t)dt$$

$$\int_0^{\delta N(Su,Tu',kt)} \varphi(t)dt < \int_0^{n(u,u',t)} \phi(t)dt \leq \int_0^{\delta N(Su,Tu',t)} \phi(t)dt$$

Which yields Su =Tu . i.e., u= u'.

Thus, A, B, S and T have unique common fixed point.

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